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DISCUSSION OF ANALYSIS OF ARCH DAMS OF VARIABLE THICKNESS

(Published in May, 1952)

By Alfred L. Parme; L. J. Mensch; Fairfax D. Kirn and Gurmukh S. Sarkaria; A. C. Josephs; George E. Goodall; and W. A. Perkins

POWER DIVISION

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DISCUSSION

ALFRED L. PARME,[8] A. M. ASCE.—The merits of arch dams with arches of variable thickness have been brought to the forefront once again in the author's excellent paper. Not only does the arch of variable thickness result in a more well-balanced stress condition than does an arch of constant thickness, but such arches simplify the task of satisfying stress requirements in the lower regions of a dam. For a given chord width, rise, and abutment thickness, the flexural stresses in an arch of variable thickness are less than those in an arch of constant thickness. Consequently, the avoidance or minimization of tension in an arch dam formed with such arches is much less of a problem than it is in one built of arches of constant thickness.

The mathematical solution presented by the author is a considerable improvement over lengthy arithmetical summations. More than this, however, the thickness relationship introduced in the paper indicates that an exact expression for the variation in thickness is not required in the elastic equations. In a published paper[9] the writer presented a similar solution for dams with symmetrical arches of variable thickness. The outline of the arches treated by the writer was composed of two circular arcs struck from different centers. The variation in thickness for these arches was shown to be given closely by the expression:

$$t = \frac{1}{1 - \cos \phi_1} [k - \cos \phi_1 - (k - 1) \cos \phi] t_o \quad \ldots\ldots\ldots (75)$$

in which k equals the ratio of the abutment thickness to the crown thickness. Also, the analysis was made on the basis that the water load was acting on the extrados face. In spite of these two noticeable differences, it is quite surprising to find that the values computed by means of Mr. Perkins' graphs agree remarkably well with values taken from the charts previously prepared by the writer.[9] For example, values interpolated from the writer's charts for $k = 1.2$ and $k = 1.3$, $\frac{t_o}{r} = \frac{24}{180} = 0.133$, and $\phi_1 = 58° 10'$ yield, for a 10-ft head, the following stresses:

Stressed fiber	Stress, in pounds per square inch
Crown extrados	47.5
Crown intrados	17.0
Abutment extrados	3.1
Abutment intrados	50.5

NOTE.—This paper by W. A. Perkins was published in May, 1952, as *Proceedings-Separate No. 129*. The numbering of footnotes, equations, illustrations, and tables in this Separate is a continuation of the consecutive numbering used in the original paper.

[8] Structural Engr., Portland Cement Assn., Chicago, Ill.

[9] "Arch Dams with Arches of Variable Thickness," by A. L. Parme, in "Modern Developments in Reinforced Concrete," Portland Cement Assn., Chicago, Ill., 1947.

Hence, for a 140-ft head, the stresses are as follows:

Stressed fiber	Stress, in pounds per square inch
Crown extrados	665
Crown intrados	238
Abutment extrados	43
Abutment intrados	710

A comparison of these values with those shown under the heading, "Numerical Example," step (7), reveals only minor differences.

This favorable agreement also applies to an arch having a small central angle, a large value of k, and relatively great thickness. As evidence of this,

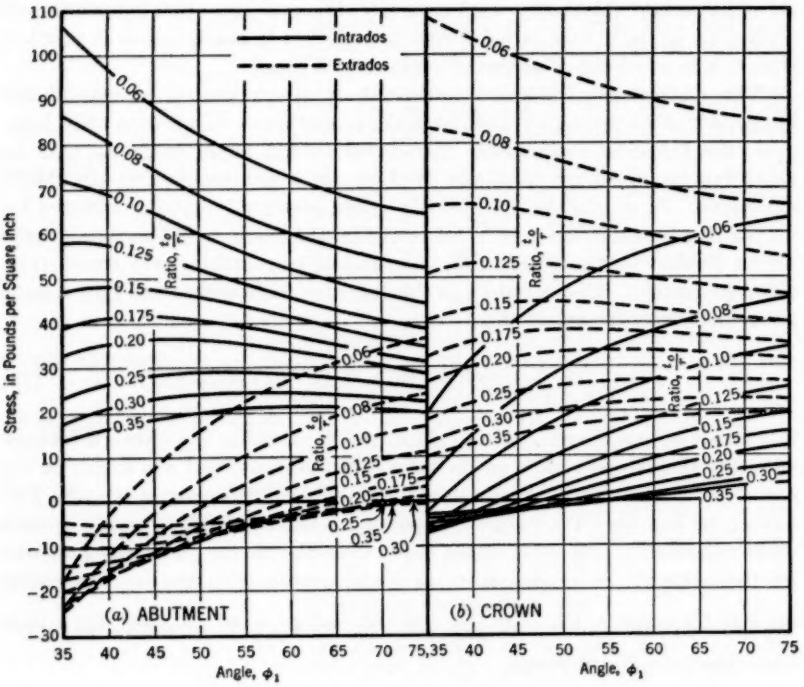

FIG. 13.—STRESSES, FOR $k = 1.5$

the stresses caused by a 10-ft head obtained by the author and previously by the writer[9] for an arch in which $\phi_1 = 35°$, $k = 1.5$, and $t_o/r = 0.35$ or $r/t_o = 2.86$.

Following the procedure outlined by the author (see under the heading, "Numerical Example"):

(1) Enter Fig. 9 with values of $t_o/t_1 = 0.667$ and $\phi = 35°$. The value of a is found to be 1.38.

(2) From Fig. 10, $t' = 1.15\,t$.

(3) Fig. 2 yields a value of K equal to 1.023. From Fig. 3, $Y = 0.0220$ and from Fig. 4, $(F + 4J)$ is found to equal 1.295.

(4) Consequently, $p\,r - P_o = \dfrac{p\,r \times 1.023}{2.862 \times 0.0220 + 1.295} = 0.694\,p\,r.$

(5) Hence, $P_o = 0.306\,p\,r$ and $P_1 = p\,r - (0.694 \times 0.819)\,p\,r = 0.432\,p\,r.$

(6) Also, by means of Fig. 5 with $B/2\,C = 0.956$: $M_o = 0.694\,p\,r^2\,(0.956 - 1) = -0.031\,p\,r^2$, and, similarly, $M_1 = 0.694\,p\,r^2\,(0.956 - 0.819) = 0.095\,p\,r^2$.

(7) With $p = 625/144 = 4.34$ lb per sq in. and $r/t_o = 2.86$, the unit stress at the crown is $s_o = 4.34 \times 2.86\,(0.306 \pm 0.031 \times 6 \times 2.86) = 3.8 \pm 6.6$. At the abutment, $s_a = 4.34 \times \dfrac{2.86}{1.5}\left(0.432 \pm \dfrac{0.95 \times 6 \times 2.86}{1.5}\right) = 3.6 \pm 9.0.$

In Table 2, the stresses obtained from Fig. 13—which is reproduced from the writer's paper[9]—are compared with the stresses obtained by the foregoing computations.

TABLE 2.—COMPARISON OF RESULTS OBTAINED BY TWO DIFFERENT METHODS

Means by which results were obtained	Stress, in Pounds per Square Inch			
	Crown		Abutment	
	Extrados	Intrados	Extrados	Intrados
(1)	(2)	(3)	(4)	(5)
Eq. 49	10.4	−2.8	−5.4	12.6
Fig. 13	10	−3	−4.8	13.2

The close agreement is not only an indication as to the validity of the author's approximation, but also tends to confirm the correctness of the algebraic computations.

Although the method derived by the author for determining the centers of the extrados and intrados arcs is correct, an alternate method involving a more direct answer is presented here. From the triangle abc, in Fig. 14, by the law of cosines—

$$r_e^2 = (r + 0.5\,t_1)^2 + X_e^2 + 2\,X_e\,(r + 0.5\,t_1)\cos\phi_1 \quad \ldots\ldots\ldots (76)$$

but, because $r_e = r + 0.5\,t_o + X_e$, it follows that—

$$(r + 0.5\,t_o)^2 + X_e^2 + 2\,X_e\,(r + 0.5\,t_o) = (r + 0.5\,t_1)^2 + X_e^2 + 2\,X_e\,(r + 0.5\,t_1)\cos\phi_1.$$

Therefore,

$$X_e = \frac{(2\,r + t_1)^2 - (2\,r + t_o)^2}{4\,[(2\,r + t_o) - (2\,r + t_1)\cos\phi_1]} \quad \ldots\ldots\ldots (77)$$

and, in a similar manner,

$$X_i = \frac{(2\,r - t_1)^2 - (2\,r - t_o)^2}{4\,[-(2\,r - t_o) + (2\,r - t_1)\cos\phi_1]} \quad \ldots\ldots\ldots (78)$$

The brief reference to the use of slip or sliding joints at the base of an arch dam is of considerable interest (see under the heading, "Numerical Example"). There is no doubt that this type of construction reduces considerably the cantilever action occurring in the lower region of an arch dam. Even if friction caused by the weight of the dam is sufficient to prevent sliding, the absence of flexural restraint greatly reduces cantilever action. In several examples investigated, the magnitude of the load carried in the vertical direction (beam action) is reduced by more than two thirds when the condition of restraint at the base is altered from a fully restrained base to a hinged base (free to rotate but incapable of sliding). Therefore, the effect of horizontal restraint at the base has a minor influence on the proportion of load carried by vertical and horizontal action. The question of how much horizontal freedom can be expected in a sliding joint is therefore not too significant.

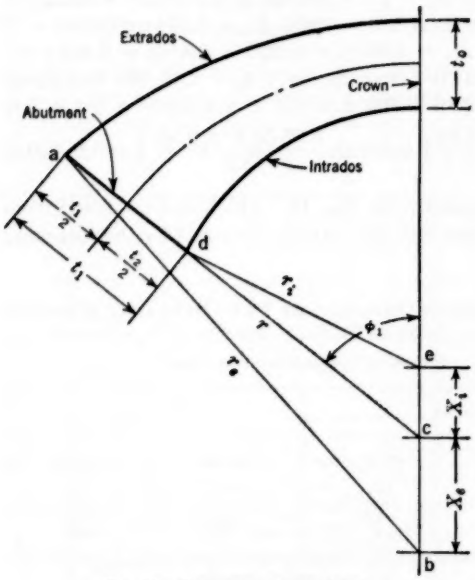

FIG. 14.—GEOMETRY OF THE ARCH

However, since this type of construction is growing in popularity, some indication of the efficacy of the sliding joint is desirable. A procedure that might yield some insight into this question is a comparison of the theoretical deflections to the actual deflection of an arch ring adjacent to the sliding joint. The interpretation of this comparison is complicated by other factors such as uncertainties in the value of the modulus of elasticity and the restraint offered by the upper rings. Nevertheless, if the author had such data, they would be valuable to the profession.

L. J. MENSCH,[10] M. ASCE.—Many years of experience have borne out the writer's repeated assertions that the solution of hydrostatic structures is more of an art than a science, and it will be shown presently that this is so in this case.

This paper treats only the case of an ideal horizontal arch slice, unaffected by its own weight, shrinkage and flow of concrete, the effect of Poisson's ratio, or the influence of adjoining arch slices.

Eqs. 40 through 45 and Eqs. 54 through 56 would have been considerably shorter if the author had selected with greater art such functions of the varia-

[10] Gen. Contractor, Evanston, Ill.

bility of t and I as to facilitate the work of integration of the differential equations. The same problem has been treated by the writer in the discussion of a paper on the same subject.[11] The writer selected in place of the author's Eqs. 1 and 2 the function:

$$\frac{1}{t} = \frac{1}{t_o}\left(1 - L_t \frac{\sin \phi}{\sin \phi_1}\right); \quad \text{and} \quad \frac{1}{I} = \frac{1}{I_o}\left(1 - L_I \frac{\sin \phi}{\sin \phi_1}\right) \ldots\ldots (79)$$

in which L_t and L_I are constants found immediately from the values of t and I assumed for the size of the arch at the crown and abutments.

In the discussion previously referred to,[11] the writer gave the following equations for an arch of uniform section, in which $\phi_1 = 60°$:

$$p\,r - P_o = \frac{p\,r}{1.33\,\frac{f^2}{l^2_o} + 1.92} \ldots\ldots (80)$$

and for an arch in which the thickness varies from $t_I = 1.5\,t_o$ at the abutment to t_o at the crown—

$$p\,r - P_o = T = \frac{p\,r}{0.707\,\frac{f^2}{l^2_o} + 1.81} \ldots\ldots (81)$$

In these equations, f denotes the rise of the arch.

The writer proposes to check the example of the author, in which $r = 180 + 12 = 192$ ft. The values of the author's example, when substituted in Eq. 80, give $p\,r - P_o = \dfrac{8{,}750 \times 192}{1.33\left(\dfrac{85.5}{24}\right)^2 + 1.92} = 89{,}000$ lb and, similarly, Eq. 81, $p\,r - P_o = 154{,}000$ lb. The correct value for the author's example should be approximately the average of 89,000 lb and 154,000 lb because $\frac{t_1}{t_o} = 1.25$ is the average of $\frac{t_1}{t_o} = 1$ and $\frac{t_1}{t_o} = 1.50$. The mean value is $\frac{89{,}000 + 154{,}000}{2}$ = 122,000 lb, which varies from the author's value by 2%. Therefore, it can be concluded that the equations suggested by the writer yield results that agree with those of the author.

FAIRFAX D. KIRN[12] AND GURMUKH S. SARKARIA,[13] J. M. ASCE.—An interesting graphical solution of an elastic, variable-thickness arch that conforms to a certain law of variation of thickness, has rigid abutments, and is subjected to uniform radial loads acting along its assumed circular center line has been presented by the author. However, it would not be technically correct to designate the method presented in this paper as "an anlysis of arch dams of variable thickness." The limited scope of this graphical solution—and hence

[11] Discussion by L. J. Mensch of "Analysis of Arch Dams by the Trial Load Method," by C. H. Howell and A. C. Jaquith, *Transactions*, ASCE, Vol. 93, 1929, p. 1250.

[12] Head, Trial Load Analysis Group, U. S. Bureau of Reclamation, Denver, Colo.

[13] Asst. Design Engr., Punjab Irrig. Branch, Simla, India.

its restricted applicability to stress analysis of arch dams—is evident from the assumptions made by the author regarding the shape of the arch ring, the rigidity of the abutments, and the characteristics of the load on the arch. Some of these assumptions are discussed here with reference to the practice of the Bureau of Reclamation (USBR), United States Department of the Interior, for the design and analysis of arch dams.

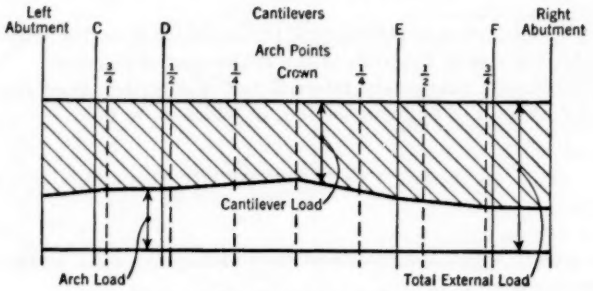

FIG. 15.—LOAD DISTRIBUTION ON AN ARCH ELEMENT

The law of variation of thickness of the arch (see Eq. 1) establishes a so-called equivalent arch for the true arch ring in an ideal arch dam of variable thickness. This equivalent arch with a circular center line is assumed to have the same properties and to behave (as a part of the homogeneous structure) in the same manner as does the true arch ring. The results of the analyses of the two arch rings, appearing subsequently in Table 3, clearly show

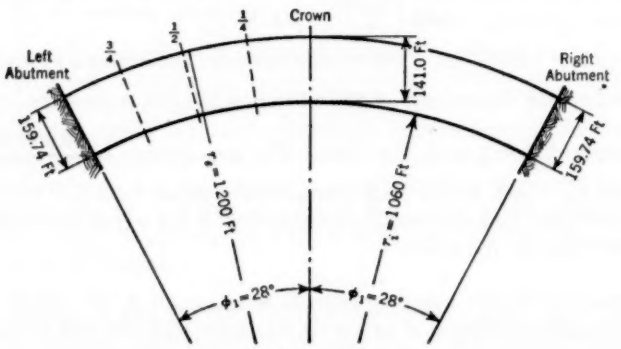

FIG. 16.—ARCH RING

that the equivalent arch ring cannot be substituted for the true arch ring of a variable-thickness arch dam. Also, the writers feel that it would be nearly impossible to lay out an arch dam composed of arch rings with circular center lines but with noncircular extradosal and intradosal faces. From the viewpoints of construction, form-work economy, and appearance, the upstream and downstream faces of the arch dam should each have a smooth and uniform

variation in vertical, as well as horizontal, curvature; and, above all, the surface should be easy to lay out at the site. Except for arches of uniform thickness, the most suitable type of arch dam from the construction standpoint has circular extradosal and intradosal faces, and has arches that vary in thickness from their crowns to their abutments. However, in many cases a dam with fillet arches will be found to be the most economical of the variable-thickness types.

Engineers who are proficient with detailed analysis of arch dams by the trial-load method recognize the importance of tangential and twist effects for determining stresses in such dams. Hence, no stress analysis of an arch dam can be considered complete or representative of actual conditions, unless proper consideration is given to the magnitude of external loads transferred to the abutments in the form of twisting moments and tangential shears, as well as vertically to the foundations by cantilever action. Thorough trial-load analyses also indicate that the radial load taken up by an arch ring of a dam does not usually have a uniform distribution from the crown to the abutments of the arch. Fig. 15 shows the load distribution on an arch element determined from a trial-load analysis of an arch dam. The head of water acting at the elevation of the arch ring is 210 ft. Radial contraction joints in the dam were assumed to be grouted, and the analysis included the effects of tangential shear and twist action, horizontal earthquake, yielding abutments, and temperature changes. Inspection of the load diagram shows that the major portion of the total load is carried by cantilever action, not by arch action, and that the load carried by the arch ring is not uniform from its crown to the abutments.

The arch ring analyzed by the author is assumed to have rigid abutments, with no consideration given the effects on the stresses in the arch produced by the yielding of the abutments or by the value of the modulus of elasticity of the abutment rock. The trial-load method of analysis recognizes and takes into consideration the effects of elastic abutments and foundations.

Fig. 16 shows one of the variable-thickness arch rings of the arch dam mentioned herein. This arch has been analyzed for the following three conditions: (*a*) As an equivalent arch ring, using the assumptions and method given in the paper under discussion; (*b*) as a true variable-thickness arch ring, assuming rigid abutments and uniform application of the water load at the extrados of the arch; and (*c*) as a true variable-thickness arch ring with the effects of abutment yielding included, and with uniform application of the water load at the extrados of the arch. (Dimensions, radii, and the central angle of the arch are shown in Fig. 16.)

From the dimensions of the variable-thickness arch as built, the dimensions for the author's equivalent arch obtained by the procedure described in his paper are as follows: r is the radius of the center line of the arch, equal to 1,129.5 ft and a is the factor used in the expression for the law of variation of thickness, equal to 0.995. This equivalent arch, when analyzed by the author's method for a water load of 13,125 lb per sq ft, assumed to be acting uniformly at the circular center line, yielded the results listed in Table 3.

In Table 3 results obtained by the author's method are compared with the stresses found by analyzing the arch shown in Fig. 16 by the method used by

the USBR for computing moments, thrusts, shears, and stresses in any arch ring.[14]

For the case of yielding abutments, the sustained modulus of elasticity of abutment rock in compression or tension was assumed to be equal to that of the concrete used in the arch dam.

A study of the results in Table 3 indicates that: (*a*) The equivalent arch designed by the author's method will have higher stresses on both faces—at the crown as well as at the abutments—than will the variable-thickness arch shown in Fig. 16. (*b*) The stresses show a general tendency to decrease for the case assuming yielding abutments. (*c*) The influence of the yielding of abutments is appreciable on stresses at the abutments. The tensile stress at the extrados of the abutment shows a decrease of 25.6% for the case assuming yielding abutments below the corresponding stress for the case of the true variable-thickness arch with rigid abutments. This decrease is 35.3%, when compared to results for the equivalent arch. (*d*) An arch of these dimensions will not be safe from the viewpoint of stresses, if it is designed to take the entire water load. The stresses obtained in all the three cases discussed here

TABLE 3.—COMPARISON OF STRESSES FOUND BY DIFFERENT METHODS[a]

METHOD	CROWN STRESSES, s_0, IN POUNDS PER SQUARE INCH		ABUTMENT STRESSES, s_1, IN POUNDS PER SQUARE INCH	
	Intrados	Extrados	Intrados	Extrados
That of the author	−269	+1,057	+1,560	−785
That of the USBR, for arch rings:				
(*a*) Assuming rigid abutments	−216	+1,001	+1,456	−683
(*b*) Assuming yielding abutments	−200	+1,046	+1,327	−508
The trial-load method	+ 88	+ 311	+ 290	+125

[a] Plus signs indicate compression; minus signs indicate tension.

are too high to be permissible in a safe structure, and indicate thickening of the arch to lower the stresses.

To clarify this last point, and to show the significance of vertical cantilever action as emphasized in the trial-load method of analysis, the stresses in this arch obtained by such an anlysis are shown in the last line of Table 3.

Fig. 15 shows the distribution of external loads between the arch and cantilever elements at this particular elevation. An examination of these results, and a comparison of them with the results of the three arch analyses described before, clearly indicate the fallacy of designing an arch dam based on the analysis of arch rings alone. The stresses obtained by trial-load analysis are within permissible limits, indicating a safe and stable structure.

In his summary, the author asserts:

> "The [arch] loads may be modified somewhat by the vertical beam or so-called 'cantilever action' of the dam, but, if the arch is designed to take full water and temperature loads, there need be no concern about the cantilever action."

[14] "Trial Load Method of Analyzing Arch Dams," *Bulletin No. 1*, Part V, Boulder Canyon Project Final Reports, U. S. Bureau of Reclamation, Denver, Colo., 1938.

Actual model tests[15] have shown that the so-called "cantilever action" exists in every arch dam, and that the water load carried by any arch element of the dam is not uniform from the crown to its abutments. The magnitude of cantilever action, or the portion of the water load that is carried by a cantilever at a certain section depends on the shape of the canyon, the location of the cantilever, and the stiffness of the cantilever concerned. An arch dam with grouted contraction joints is a homogeneous structure. Therefore, wishful thinking will not prevent the vertical cantilevers from transferring part of the external load to the foundation by cantilever action. The division of load between the two systems of elements must be such as to maintain continuity of deformation and rotation throughout the structure. In any case, if the arches in an arch dam are designed to resist the full water load, temperature effects, and other stresses, with no consideration being given to the cantilever action, the arch rings will be unnecessarily oversafe and uneconomically designed. Also, the computed stresses will be incorrect. Even the provision of a horizontal sliding joint at the base of the arch dam cannot completely nullify the vertical cantilever action because the vertical elements will still transfer a part of the external load to its base by horizontal shears acting at the sliding joint. Therefore, it is difficult to understand how a complete stress picture can be obtained for any arch dam by neglecting cantilever action and by using the author's method of arch ring analysis.

Although it is true (as the author states) that in several published articles, for ease of computation, the vertical beams have been assumed to have parallel faces—thus giving undue credit to stiffness of the cantilevers—this statement does not apply to the trial-load method of analysis as used by the USBR in the design of arch dams. In this method of analysis, the vertical cantilever elements are assumed to have radial sides (not parallel sides); and all computations are based on this assumption.

The trial-load method makes it possible to analyze load distributions, deflections, and stresses in curved concrete or masonry dams of all sizes and shapes—whether of the massive arch gravity type, the thin arch type, or the cupola-shaped design. This method considers the deformation of the abutments and foundation, the shape of the canyon in which the dam is to be built, and the construction program planned for grouting any contraction joints in the dam. Furthermore, the trial-load method includes the effects of tangential shear and twist action that exist in every dam. These cannot be determined by the author's method of analysis. Also, the correctness of the trial-load method has been substantiated by tests on models[15] as well as the Stevenson Creek Test Dam[16] in California.

In defense of the trial-load method of analysis—if understood, this method is not unduly cumbersome, nor is the length of time required to make an analysis unreasonable. High concrete dams are costly structures; and their structural behavior is too complicated to be interpreted and analyzed by oversimplified short-cut methods. The sum of money that can often be saved by determining the true picture of stresses in such a structure by using a thorough

[15] "Model tests of Boulder Dam," *Bulletin No. 3*, Part V, Boulder Canyon Project Final Reports, U. S. Bureau of Reclamation, Denver, Colo., 1939.

[16] "Arch Dam Investigation," Vol. I and Vol. II, The Eng. Foundation, New York, N. Y., 1934.

and complete method of analysis (such as the trial-load method), and the evolution of a safe and satisfactory design, are two important factors in favor of comparatively lengthy but more thorough analyses. Simplified and abbreviated methods based on drastically limiting assumptions, such as the author's method, have a definite value in the preliminary design of arch dams. However, such methods should not be used as the final analyses for the design of high arch dams.

A. C. JOSEPHS,[17] A. M. ASCE.—The use of design curves, pattern loads, tables of constants, and other labor-saving aids has become necessary for the analysis of most arch dams if the engineer is to have any time available to make the necessary design studies. The mechanics in the analysis of the elements of the dam have received considerable attention; actual design has been dictated by, or limited to, forms for which simple methods of analysis are available. As a result, there has been little or no progress made in the art of design of this structure since about 1932. The writer will confine his discussion to a few brief comments on some aspects of design.

In the "Summary," the author states: "For any given site, the arch dam with the minimum volume of concrete will be of the variable radius type." If this statement is presumed to mean that the horizontal arch elements are essentially circular and that the radii vary with the elevation, this generalization does not seem to be justified. This criterion for minimum volume has wide acceptance and would probably be correct if each arch ring were free to carry its load by deflecting independently of its neighbors. The arch rings are not free to deflect in such a manner, and the restraining influence of the adjacent rings and of the mass forces causes a "redistribution" of the load between the various arch elements. The shape of the redistributed load is a function of the shape or profile of the dam site.

Designers who have had the experience of analyzing arch dams by the "trial-load method"[18] are aware that once an approximate agreement between the deflections of the arch and gravity elements is reached, changing the volume or shape of the dam within practical limits does not appreciably alter the shape of this redistributed load. Very rarely will the load distribution between arch rings result in a load that is uniform along any horizontal arch ring, and in the case of sites which are irregular or unsymmetrical, the load will not even approximate that condition—whether or not the mass forces are considered. One of the rewards of the long and tedious effort of a trial-load analysis is the insight gained by the designer into the individual characteristics of that particular site. If the dam is shaped to conform to this loading, there will be a minimum of load transfer between the arch rings, and an economical design that conforms to the restraint characteristics of the site will result. An irregular or unsymmetrical site will normally require arch rings of compound curvature. The writer made preliminary studies[19] of a design along

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[18] "Analysis of Arch Dams by the Trial Load Method," by C. H. Howell and A. C. Jaquith, *Transactions*, ASCE, Vol. 93, 1929, p. 1191.

[19] "Ueber den Entwurf gewölbte Mauern," Technische Hochschule, Vienna, Austria, 1933.

this line about 1933, but had neither the staff nor the time to bring it to its logical conclusion—that is, the complete analysis of a dam so designed.

In the writer's opinion, an arch dam of minimum volume will result if the arch rings carry their load with a minimum of restraint. In all cases known by the writer where the design investigation has been thorough, the inclusion of the gravity effect in the design calculations has always resulted in a very substantial increase in the volume of the dam because the gravity element restrains the arch and prevents it from carrying its load effectively. The gravity or mass effect does exist and the designer must take it into consideration. For arch dams of moderate height, located on favorable sites, the experienced designer can, and should, design the dam for maximum economy, as outlined in the summary of Mr. Perkins' paper. However, to insure a safe and economical design, major structures and those at "unfavorable" sites should have the benefit of a detailed study of the effects of restraint.

GEORGE E. GOODALL,[20] M. ASCE.—Without having checked the derivation of the various formulas in this paper, the writer can testify, nevertheless, to the accuracy of the results that they yield. In 1946, an existing arch dam was investigated to determine the feasibility of raising it. The thickness toward the abutment was increased by shifting the center of curvature of the

TABLE 4.—COMPARISON OF UNIT ARCH STRESSES, IN POUNDS PER SQUARE INCH

Surface	ABUTMENT[a]			CROWN		
	Perkins	Goodall	Vogt	Perkins	Goodall	Vogt
Intrados	456	452	562	24	64	57
Extrados	−96	−78	−16	440	414	382

[a] Minus sign indicates tension.

intrados upstream from the center that was used for the extrados along the symmetry of the ring. The arch thus constructed is a close approximation of the arch envisioned by "the law of variation of thickness" (Eq. 1).

For example, consider an arch ring with thicknesses of 40 ft and 60 ft at the crown and abutment, respectively. Radii r_e and r_i were 240.0 ft and 148.9 ft; $2\phi_l$ was 106° 46′, and the design head was 135 ft. In 1946 the writer applied the methods of A. Floris[21] to this arch, and later compared the results by Mr. Perkins' procedure. Table 4 indicates that stresses computed by either method are sufficiently accurate for design purposes and that Figs. 4 to 10 reduce the work to a minimum for this type of arch. Both methods of analysis disregard the effects of the displacement of the neutral axis, Poisson's ratio, and deformation of the abutments. Some approximate calculations of stresses where moments and thrusts are referred to the neutral axis instead of the gravity axis indicate that disregard of the true location of the neutral axis is of minor

[20] Cons. Engr., Sacramento, Calif.

[21] Discussion by A. Floris of "Analysis of Arch Dams by the Trial Load Method," by C. H. Howell and A. C. Jaquith, *Transactions*, ASCE, Vol. 93, 1929, p. 1296.

importance as compared to the effects of lateral strains and deformation of the abutments. This fact has been well illustrated by Fredrik Vogt,[22] M. ASCE.

As a matter of fact, an arch with a uniform thickness of 40 ft, having the same inside radius and central angle as the foregoing, and under the same head, has 562 lb per sq in. and −16 lb per sq in. at the abutment intrados and extrados, respectively, and 57 lb per sq in. and 382 lb per sq in. for the corresponding values at the crown (Table 4), when analyzed by the Vogt formulas, which include the effects omitted by Mr. Perkins. A very small increase in either thickness or central angle would eliminate the tensile stress, and the maximum compressive stress would still be low.

It has been the writer's practice to include these effects in the design of arch dams. By taking advantage of the known effects of the lateral strains and abutment yield, it is possible to design a much more economical arch dam for a given site, within allowable stresses, than by increasing abutment thickness, as proposed by the author.

In the closing paragraph of the paper the author states:

> "* * * the controlling height of an arch dam, within the limits of practical dimensions and of allowable unit stresses and without excessive tensile stresses at the extrados near the abutments, is between 300 ft and 400 ft."

It is suggested that the author give his thoughts on what maximum unit compressive stresses should be allowed in the arches of dams built with good concrete on sound rock. The writer contends that no tensile stresses should be permitted in arch dams. Every practical arch dam of which the writer has any knowledge was constructed with a number of contraction joints. If tension is permitted at the extrados near the abutments of the arches, there is certain to be some place along the profile where this tensile stress would be indicated across a contraction joint which obviously can transmit no tensile stress. Furthermore, the writer does not place any reliance in the tensile stress that can be developed between the concrete of the arch and the rock abutment against which it has been placed.

With the invention of the vibrator and the development of modern weigh-batching plants, it has been demonstrated repeatedly that concretes having a twenty-eight-day crushing strength of 4,000 lb per sq in. are readily attained. Since the lower parts of an arch dam will generally have aged six months or more prior to loading, it would seem that compressive stresses of 1,000 lb per sq in. in compression are conservative. However, no tensile stresses should be permitted. If the effects of lateral strains and deformation of abutments are included in the arch analysis, the limiting height, as determined by the writer, can be greatly exceeded. In the design of an arch dam to be constructed in California to a height of nearly 480 ft above its foundation, all arch rings have been analyzed by the Vogt formulas. They were shaped so as to have no tensile stresses; yet the maximum compressive stress is slightly less than 850 lb per sq in. However, this result would have been impossible had the effects of lateral strains and deformation of the abutments been neglected.

[22] Discussion by Fredrik Vogt of "Stresses in Thick Arches of Dams," by B. F. Jakobsen, *Transactions*, ASCE, Vol. 90, 1927, p. 561.

Acknowledgment.—The writer acknowledges the assistance of Jack S. Barrish, A. M. ASCE, and Klyne G. Beaumont, J. M. ASCE, who made and checked some of the computations entering into this discussion.

W. A. PERKINS,[23] M. ASCE.—In order to provide a ready means of analyzing stresses caused by water load and temperature changes in a horizontal slice of an arch dam of varying thickness, and to determine the deflection at the crown, this paper was written. As it would be of little practical value without the curves, it was not presented for publication until these curves could be completed. During the interval an excellent treatise was published by Mr. Parme.[9] Since this paper did not make provision for determining temperature stresses, and since the method of approach was different, the writer concluded that, for the benefit of those familiar with Mr. Cain's paper, and to determine the effects of temperature, publication of this paper was justified.

The writer questions Mr. Parme's reference to the "author's approximation." The formulas were developed in accordance with the principle of least work, which is an exact, and not an approximate, principle. The only approximation made is in assuming that the load P_0 acts radially on the center line of the dam, which is not exactly true, as was explained in the paper.

Mr. Parme inquired about the sliding joint referred to in the paper. At the Matilija Dam in Ventura County, California, one of these joints was placed 4 ft above the base at the center line. This dam is in a U-shaped canyon, and at the elevation of the joint the dam is approximately 200 ft long. The arch deflection at this height, 154 ft below the crest, would be about ¾ in. without restraint. Such a condition would develop very high shear stresses at the foundation contact and introduce indeterminate stresses.

The joint was constructed by carefully leveling and finishing the surface at the joint level. This was then covered with a layer of a commercial, highly compressed, graphite-impregnated, asbestos planking. The planking was covered with a thin coating of graphite that, in turn, was covered with paper, and the upper layers of concrete were poured thereon. Leakage was prevented by a copper water stop bolted to the upstream face of the dam and this, in turn, was protected by a steel plate.

This reservoir filled for the first time in the fall of 1952, and there was opportunity for observing the behavior of the joint. It was found that at the crown the deflection was 1.02 in. and at the quarter points, 0.59 in. and 0.73 in., respectively. The deflection of 1.02 in. instead of ¾ in. occurred because the ¾-in. deflection did not include temperature effects and, also, there is probably some abutment yield.

The joint installed in 1930 was observed by the writer from 1939 to 1950, and it was found to act according to expectations. This joint is at approximately mid-height of the dam, at which the head is 50 ft. It was installed at the elevation where the right abutment changed from approximately vertical to approximately horizontal, and conditions made it necessary to extend the arch for a considerable distance along this bench to a point where it was

[23] Cons. Engr., Sacramento, Calif.

feasible to construct a gravity thrust block. A movement of about ⅜ in. has been observed, and a larger movement is not expected. The use of these joints, with gravity sections below, will be found very advantageous in the design and construction of arch dams.

It is interesting to observe the variety of opinions among engineers regarding the merits of the trial load method for the analysis of dams. This can be noted in the discussions of this paper, some of which advocate exclusive use of this method, whereas others ignore it. This same divergency of opinion will be found among engineers in general who have given this subject careful attention

This divergency results from two different approaches to the problem. One approach is to assume that the arch should be designed to take full water load, the other, that the cantilever take the major portion. Designs which attempt to divide the load between arch and cantilever action frequently result in structures that show high tensile stresses both in the arches and at the base of the cantilever.

At a site that is suitable for an arch, it will be found that a thin arch designed to take the full water load without tension will require much less concrete than a structure that is designed to carry a part of the load by gravity action. Such an arch, if of considerable height, or if the site is irregular, or if there are abrupt breaks in the profile of the abutments, should be investigated for vertical beam action so as to determine the stresses at the foundation and the load on the mid-arches in the vicinity of the breaks. In some cases, it has been found that under such conditions the arches were carrying a load in excess of the direct water load.

For dams of heights of from 300 ft to 400 ft, analysis by the trial load method is an expensive diversion. For extremely high dams, such as Hoover Dam, on the Colorado River in Arizona and Nevada, all precautions of design should be utilized. Hoover Dam, however, is essentially a gravity structure as its base thickness is approximately 90% of its height. Because of its height it is estimated, without any study, that the deflection of the crest under full load is approximately 3 in. or 4 in., which would produce considerable arch resistance in the upper section since it is curved in plan. Terming such curved gravity dams "arch dams" is a misnomer and is probably the chief reason for disagreement among engineers regarding the methods of an analysis to be used for true arches.

With reference to Fig. 16, in the discussion submitted by Messrs. Kirn and Sarkaria, the dam depicted is termed an arch—but it will be noted that the crown thickness is 141 ft, or 67% of the height to the crest, and at the abutments it is 159.74 ft, or 76% of the height. Such measurements would classify this dam as a gravity section. The section shown in Fig. 16 could be replaced by a true arch with a crown thickness of approximately 90 ft, an abutment thickness of 120 ft, a radius of 595 ft, a central angle of 125°, and an average thickness of approximately 98 ft. The difference in thickness is approximately 50 ft from that proposed by Messrs. Kirn and Sarkaria, allowing for a large saving in concrete.

The comparison of stresses in Table 3 is of little value since no one would think of designing a true arch of the dimensions shown. From their remarks about construction, it would appear that Messrs. Kirn and Sarkaria did not read the last part of the paper embracing Eq. 67 to Eq. 74.

Inclusion of tangential and twist effects in the analysis of an arch dam will cause a reduction of stresses below those found when these are not included. Their inclusion would add two more terms to the right side of Eq. 13, thus reducing the amount of work done by each of the three terms used therein. The tangential and twist effects are relatively small, however, and in moderate size dams will not indicate any appreciable saving in the quantity of concrete in the dam.

Messrs. Kirn and Sarkaria and Mr. Goodall have stressed the importance of abutment yield. Little is known about the true behavior of the abutments and foundations of dams. Therefore, in order to evaluate the results of this part of the stress analysis of dams, it is necessary to investigate the validity of the assumptions that are made concerning this behavior. In this closure, the remarks will be confined to arch dams, but the same general principles apply to gravity dams.

It may be logically assumed that elastic yield is the only type of yield that may be mathematically analyzed. The arch action ends at the contact with the abutment at the termination of the section where the value of the moment of inertia used in the arch analysis is applicable. Abutment yield can best be divided into two parts—translation and rotation. Translation consists of the elastic yield of the abutment caused by pressure on the rock throughout the width of the concrete contact, equal to the pressure at the extrados. Rotation is the yield caused by the additional stress at the intrados.

The yield of translation will have the same effect as a drop in temperature. The amount of this yield will be $\frac{s_e l}{2 E_r}$, in which s_e is the unit stress at the extrados, l is the distance to the point of no yield in the rock, and E_r is the modulus of elasticity of the rock. In the arch a drop of t degrees will cause a shortening equal to $0.0000055\ t\ r\ \phi$. To determine the arch stresses, it is necessary to find a value of t equivalent to the amount of abutment yield. From the equation,

$$t = \frac{s_e l}{(2 E_r r \phi)\ 0.0000055} \quad \ldots\ldots\ldots\ldots\ldots\ldots (83)$$

t can be determined and the stresses can be found with the aid of Eq. 59*a* and Eq. 59*b*.

Rotation at the abutment will cause a tendency for rotation of the crown, and the resistance to this rotation, by equal and opposite reactions from the other half arch, causes stresses, induced by moment, which act in the same direction as those resulting from water loading. These stresses, however, cause a resistive moment at the abutment which increases the compressive stress at the extrados and decreases it at the intrados.

A method for determining the moments and stresses caused by the rotation due to abutment yield will be presented. This method is an approximation, but, since the stresses involved are not large, the results should be made within the limits of basic assumptions. The yield of the rock y_r equals $\frac{(s_i - s_e)\, l}{2\, E_r}$ as in translation. The yield of the concrete at the crown will be

$$y_c = \left[\frac{(s_i - s_e)\, l}{2\, E_r} \times \frac{t_o}{2\, t_1}\right] = \frac{s_c\, r\, \phi}{E_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

Since the rotation at the crown produces both compression and tension, the ratio of stresses at the abutment and crown is $\frac{t_o}{2\, t}$.

$$s_c = \frac{E_c}{2\, E_r}\,(s_i - s_e)\,\frac{l\, t_o}{r\, \phi\, 2\, t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (85)$$

and

$$M = \frac{s_c}{2} \times \frac{4\, t_o}{3} = \frac{E_c}{6\, E_r}\,(s_i - s_e)\,\frac{l\, t_o}{r\, \phi\, t_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (86)$$

This moment acts at the crown in the same direction as the moment due to water load, but at the abutment it acts in the opposite direction so as to reduce the moment and stresses at that point. These moments are combined with the load moments to find the unit stresses.

Mr. Goodall made a comparison of stresses in an arch of varying thickness as computed by his and the writer's methods. However, the studies were not carried far enough to reveal the advantages of an arch of varying thickness as indicated by the following example: An arch slice was computed lying entirely within the area embraced in the 40-ft constant thickness design suggested by Mr. Goodall, with these assumptions and results: Abutment at intrados and crown at extrados coincident with like points in design; $t_o = 24$ ft, $t_1 = 32$ ft, $a = 0.67$, $t' = 26.33$ ft, $2\,\phi = 121° 54'$, and $r = 151.22$ ft. The stresses were as follows: Extrados—crown stress, 556 lb per sq in., and abutment stress, 10 lb per sq in.; intrados—crown stress, 175 lb per sq in., and abutment stress, 565 lb per sq in. The close balance between the maximum compressive stresses at the crown and at the abutment indicates the most economical use of concrete in the arch. The maximum stresses in the two arches are almost identical; yet the average thickness of this arch is less than two thirds of the 40-ft uniform thickness arch recommended by Mr. Goodall, and there is no tension at the abutment.

Mr. Goodall inquires about the controlling height of arch dams and the assumed allowable stress. Permissible stress depends as much on the quality of the abutments as on that of the concrete. With a good quality of concrete mixed from good materials in a modern mixing plant, a compressive stress of 1,000 lb per sq in. is a conservative value. However, this value is too high for the quality of many of the abutments that are encountered. Since the design is usually completed before the abutments are uncovered, the limiting stress must be a matter of judgment.

With respect to the limiting height of dams, reference is made to a frequently used curve.[24] Assuming the condition of an arch with zero stress at the extrados of the abutments, and a unit stress limit of 1,000 lb per sq in., the average stress is 500 lb per sq in. By the cylinder formula, $t = \frac{p\,r}{s}$, or $p = \frac{t\,s}{r}$, so in order to avoid tension at the extrados of the abutment, with a value of $\frac{t}{r} = 0.3$, the central angle must equal 140°, an extremely high value. Solving with these values, $p = 150$ lb per sq ft and the head is equal to 345 ft. Use of the cylinder formula is a close approximation since $P_1 = p\,r - (p\,r - P_0) \cos \phi$ and $(p\,r - P_0) \cos \phi$ is only a small percentage of $p\,r$.

The discussions by Mr. Josephs and Mr. Mensch are constructive and helpful and do not call for extended comment by the writer except in the use of the sliding joint. The sliding joint is used since it will greatly aid in minimizing the restraint mentioned by Mr. Josephs.

[24] "Stresses in Thick Arches of Dams," by B. F. Jakobsen, *Transactions*, ASCE, Vol. 90, 1927, p. 511

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